1980/BC5)

a) find the general sol^n of B:

$$xy' + y = 0$$

Ans:

$$xy' = -y$$

$$x\frac{dy}{dx} = -y$$

separate variables
"isolate" variables

sol^n to a diff eq^n is a function
expressed as $y = \sim\sim$

$$\frac{x}{x}\frac{dy}{y} = -\frac{y}{y}\frac{dx}{x}$$

$$\int \frac{dy}{y} = \int -\frac{dx}{x}$$

$$\int \frac{1}{y}\,dy = -\int \frac{1}{x}\,dx$$

$$e^{(\ln|y|)} = e^{(-\ln|x| + C)}$$

$$|y| = e^{-\ln|x|} \cdot e^{C}$$

$$a^m \cdot a^n = a^{m+n}$$

$\sim + C_2 = \sim + C_1$
$-C_2 \quad -C_2$
$\sim = \sim + (C_1 - C_2)$

$$|y| = Ae^{\ln(|x|^{-1})}$$

$$A = e^C$$
$$A > 0$$

$$|y| = A\frac{1}{|x|}$$

$$y = \pm A\frac{1}{|x|}$$

1980/BC5 b)

$$xy' + y = 2x^2 y$$

$$x\frac{dy}{dx} = 2x^2 y - y = y(2x^2 - 1)$$

$$\frac{1}{y}\,dy = \frac{2x^2-1}{x}\,dx$$

$$\int \frac{1}{y}\,dy = \int \frac{2x^2-1}{x}\,dx = \int \frac{2x^2}{x}\,dx - \int \frac{1}{x}\,dx$$

$$\ln|y| = (x^2 - \ln|x| + c)$$

$$e \qquad e$$

$$|y| = \frac{e^{x^2}}{e^{\ln|x|}} \cdot e^{c} = \frac{Ae^{x^2}}{|x|} \qquad A>0$$

$$y = \pm A \frac{e^{x^2}}{|x|}$$

$$y = A\frac{e^{x^2}}{|x|},\ A \neq 0$$

1980/BC5/c

$$y = A\frac{e^{x^2}}{|x|}, A \neq 0$$

$$\underbrace{y = A\frac{e^{x^2}}{x}, A \neq 0}$$

find particular solution

$y = e^2$ when $x = 1$

$$\frac{e^2}{e} = A\frac{e^{1^2}}{1} = \frac{Ae}{e}$$

$$\therefore e = A$$

part. sol^n:

$$y = e\frac{e^{x^2}}{x} = \frac{e^{(x^2+1)}}{x}$$

1980 BC5 a)

$xy' + y = 0$

sol'n to a diff eq'n will be a FUNCTION

$y = \dots$

$x\frac{dy}{dx} = -y$

SEPARATE variables "isolate" variables

$x\,dy = -y\,dx$

☆ $\int \frac{1}{y}dy = \int -\frac{1}{x}dx$

$\ln|y| + c_1 = -\ln|x| + c_2$

$-c_1 \quad -c_1$

$\ln|y| = -\ln|x| + (c_2 - c_1)$

$\ln|y| = -\ln|x| + C$

$e^{(\ln|y|)} = e^{(-\ln|x| + c)}$

$a^m \cdot a^n = a^{m+n}$

$|y| = e^{\ln(|x|^{-1}) + c} = e^{\ln(|x|^{-1})} \cdot e^c$

$|y| = A\frac{1}{|x|}$ $\quad A = e^C$, $A > 0$

$y = \pm A\frac{1}{|x|}$

or $y = \frac{A}{|x|}$, $A \neq 0$

Feb 23-7:59 AM

1980 BC5b)

$$xy' + y = 2x^2y$$

$$x\frac{dy}{dx} = 2x^2y - y = y(2x^2-1)$$

$$\frac{1}{y}dy = \frac{2x^2-1}{x}dx = \left(2x - \frac{1}{x}\right)dx$$

+!

$$\int \frac{1}{y}dy = \int 2x - \frac{1}{x}\,dx$$

$$\ln|y| = (x^2 - \ln|x| + c)$$

$$e \qquad e$$

$$A = e^c$$

$$A > 0$$

$$|y| = \frac{e^{x^2}}{e^{\ln|x|}} \cdot e^c = A\frac{e^{x^2}}{|x|}$$

$$y = \pm A\frac{e^{x^2}}{|x|}$$

Feb 23-11:39 AM

1980 BC5c)

$y = \pm A \frac{e^{x^2}}{|x|}$

$y = e^2$ when $x = 1$

$e^2 = \pm A \frac{e^{(1)^2}}{|1|} = e^2 = \pm A \cdot e$

$+e = \pm A$

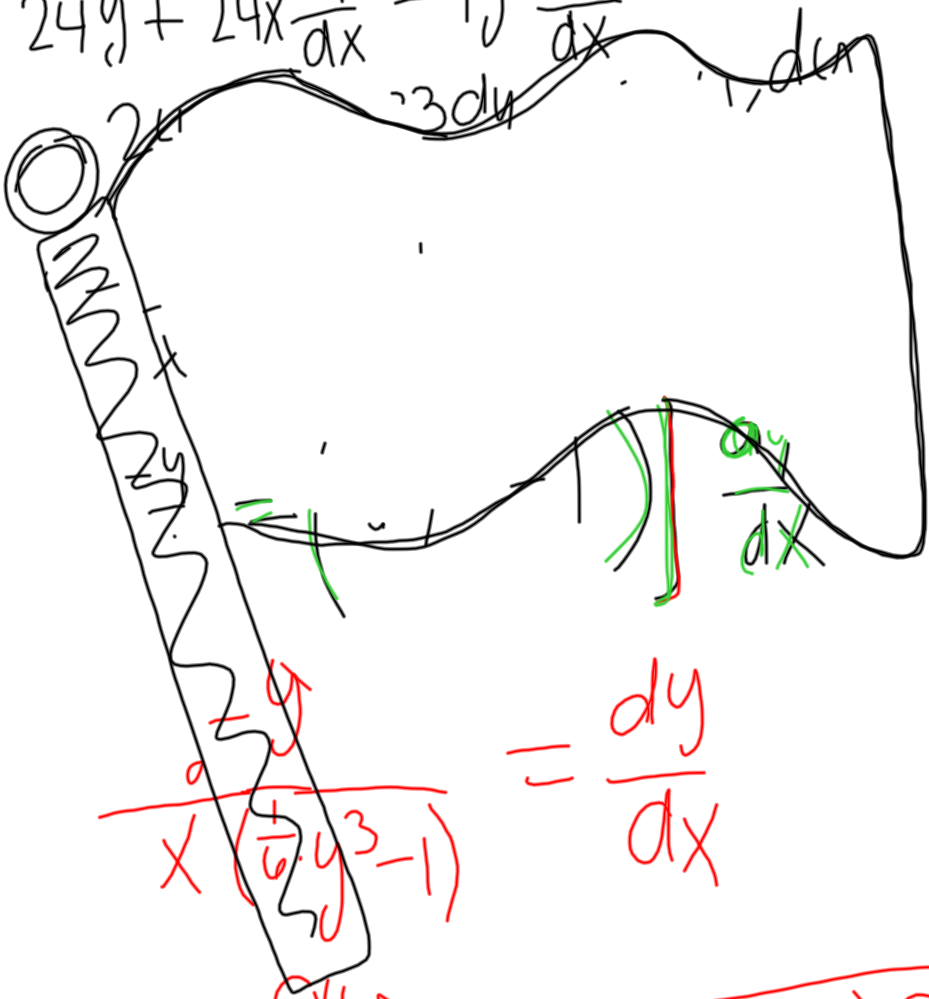
$A = e$

$y = e \frac{e^{x^2}}{|x|}$

soln is a FUNCTION with a domain

$$24xy = y^4 + 48 \qquad \sqrt{1+(f'(x))^2} \qquad y=2 \quad y=4$$

$$24y + 24x\frac{dy}{dx} = 4y^3\frac{dy}{dx}$$



$$\frac{-y}{x\left(\frac{1}{6}y^3-1\right)} = \frac{dy}{dx}$$

$$\text{arclength} = \int_2^4 \sqrt{1+\left(\frac{-y}{x\left(\frac{1}{6}y^3-1\right)}\right)^2}$$